

M2-branes and background fields

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M2-branes and background fields

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ABSTRACT: We discuss the coupling of multiple M2-branes to the background 3-form and 6-form gauge fields of eleven-dimensional supergravity, including the coupling of the Fermions. In particular we show in detail how a natural generalization of the Myers flux-terms, along with the resulting curvature of the background metric, leads to mass terms in the effective field theory.

KEYWORDS: Brane Dynamics in Gauge Theories, Extended Supersymmetry, Supersymmetric Effective Theories, M-Theory

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1 Introduction

For a single M2-brane propagating in an eleven-dimensional spacetime with coordinates x^m the full non-linear effective action including Fermions and κ -symmetry was obtained in [1]. The Bosonic part of the effective action is

$$S = -T_{M2} \int d^3\sigma \sqrt{-\det(\partial_\mu x^m \partial_\nu x^n g_{mn})} + \frac{T_{M2}}{3!} \int d^3\sigma \epsilon^{\mu\nu\lambda} \partial_\mu x^m \partial_\nu x^n \partial_\lambda x^p C_{mnp} . \quad (1.1)$$

Here C_{mnp} is the M-theory 3-form potential, g_{mn} the eleven-dimensional metric and $T_{M2} \propto M_{pl}^3$ is the M2-brane tension.

If we go to static gauge, $\sigma^\mu = x^\mu$, $\mu = 0, 1, 2$ then the M2-brane has world-volume coordinates x^μ and the x^I , $I = 3, 4, 5, \dots, 10$ become 8 scalar fields. In this paper we will be interested in the lowest order terms in an expansion in the eleven-dimensional Planck scale M_{pl} . In this case the canonically normalized scalars are $X^I = x^I \sqrt{T_{M2}}$. These have mass-dimension 1/2 whereas g_{mn} and C_{mnp} are dimensionless.

We next seek a generalization of this action to lowest order in M_{pl} but for multiple M2-branes. The generalization of the first term in (1.1) was first proposed in [2–5]. This has the maximal $\mathcal{N} = 8$ supersymmetry and describes two M2-branes in an $\mathbb{R}^8/\mathbb{Z}_2$ orbifold [20, 21] but cannot be extended to more M2-branes [6, 7] (although there are interesting models with Lorentzian signature on the 3-algebra [8, 9]). It was then further generalized in [10, 11] for arbitrary M2-branes and manifest $\mathcal{N} = 6$ supersymmetry in an $\mathbb{R}^8/\mathbb{Z}_k$ orbifold.

In this paper we will obtain the generalization of the second term (i.e. the Wess-Zumino term) which gives the coupling of the M2-branes to background gauge fields. In the well studied case of D-branes, where the low energy effective theory is a maximally

supersymmetric Yang-Mills gauge theory with fields in the adjoint representation, the appropriate generalization was given by Myers [12]. In the case of multiple M2-branes the scalar fields X^I and Fermions now take values in a 3-algebra which carries a bifundamental representation of the gauge group. Thus we wish to adapt the Myers construction to M2-branes. For alternative discussions of the coupling of multiple M2-branes to background fields see [17–19].

The rest of this paper is as follows. In section 2 we will discuss the relevant couplings, to lowest order in M_{pl} , for the $\mathcal{N} = 8$ Lagrangian of [4] and demonstrate that, by an appropriate choice of terms, the action is local and gauge invariant. We will also supersymmetrize the case where the background field G_{IJKL} is non-vanishing and demonstrate that this leads to the mass-deformed theories first proposed in [13, 14]. In section 3 we will repeat our analysis for the case of $\mathcal{N} = 6$ supersymmetry, which includes both the ABJM [10] and ABJ [11] models leading to the mass deformed models of [15, 16]. In section 4 we will discuss the physical origin of the flux-squared term that arises by supersymmetry. In particular we will demonstrate that this term arises via back reaction of the fluxes which leads to a curvature of spacetime. Section 5 will conclude with a discussion of our results.

2 $\mathcal{N} = 8$ Theories

Let us first consider the maximally supersymmetric case. We follow the notion and conventions of [25]. Although this case has only been concretely identified with the effective action of two M2-branes in an $\mathbb{R}^8/\mathbb{Z}_2$ orbifold [20, 21] it is simpler to handle and hence the presentation is clearer. In the next section we will repeat our analysis for the case of $\mathcal{N} = 6$.

2.1 Non-Abelian couplings to background fluxes

The scalars X^I live in a 3-algebra with totally anti-symmetric triple product $[X^I, X^J, X^K]$ and invariant inner product $\text{Tr}(X^I, X^J)$ subject to a quadratic fundamental identity and the condition that $\text{Tr}(X^I, [X^J, X^K, X^L])$ is totally anti-symmetric in I, J, K, L [4]. An important distinction with the usual case of D-branes based on Lie algebras is that Tr is an inner-product and not a map from the Lie algebra to the real numbers. In particular there is no gauge invariant object such as $\text{Tr}(X^I)$. Thus the only gauge-invariant terms that we can construct involve an even number of scalar fields.

In this paper we wish to consider the decoupling limit $T_{M2} \rightarrow \infty$ since, unlike String Theory, there are no other parameters that we can tune to turn off the coupling to gravity. In particular it is not clear to what extent finite T_{M2} effects can be consistently dealt with in the absence of the full eleven-dimensional dynamics.

Assuming that there is no metric dependence we start with the most general form for a non-Abelian pull-back of the background gauge fields to the M2-brane world-volume:

$$\begin{aligned}
 S_C = \frac{1}{3!} \epsilon^{\mu\nu\lambda} \int d^3x & \left(a T_{M2} C_{\mu\nu\lambda} + 3b C_{\mu IJ} \text{Tr}(D_\nu X^I, D_\lambda X^J) \right. \\
 & + 12c C_{\mu\nu IJKL} \text{Tr}(D_\lambda X^I, [X^J, X^K, X^L]) \\
 & \left. + 12d C_{[\mu IJ} C_{\nu KL]} \text{Tr}(D_\lambda X^I, [X^J, X^K, X^L]) + \dots \right) \quad (2.1)
 \end{aligned}$$

where a, b, c, d are dimensionless constants that we have included for generality and the ellipsis denotes terms that are proportional to negative powers of T_{M2} and hence vanish in the limit $T_{M2} \rightarrow \infty$.

Let us make several comments. First note that we have allowed the possibility of higher powers of the background fields. In D-branes the Myers terms are linear in the R-R fields however they also include non-linear couplings to the NS-NS 2-form. Since all these fields come from the M-theory 3-form or 6-form this suggests that we allow for a non-linear dependence in the M2-brane action.

Note that gauge invariance has ruled out any terms where the C -fields have an odd number of indices that are transverse to the M2-branes (although the last term could have a part of the form $C_{\mu\nu I} C_{JKL}$). This is consistent with the observation that the $\mathcal{N} = 8$ theory describes M2-branes in an $\mathbb{R}^8/\mathbb{Z}_2$ orbifold and hence we must set to zero any components of C_3 or C_6 with an odd number of I, J indices.

The first term is the ordinary coupling of an M2-brane to the background 3-form and hence we should take $a = N$ for N M2's. The second line leads to a non-Lorentz invariant modification of the effective 3-dimensional kinetic terms. It is also present in the case of a single M2-brane action (1.1) where we find $b = 1$ which we will assume to be the case in the non-Abelian theory.¹ The final term proportional to d in fact vanishes as $\text{Tr}(D_\lambda X^I, [X^J, X^K, X^L]) = \frac{1}{4} \partial_\lambda \text{Tr}(X^I, [X^J, X^K, X^L])$ which is symmetric under $I, J \leftrightarrow K, L$. Thus we can set $d = 0$.

Finally note that we have allowed the M2-brane to couple to both the 3-form gauge field and its electromagnetic 6-form dual defined by $G_4 = dC_3$, $G_7 = dC_6$ where

$$G_7 = \star G_4 - \frac{1}{2} C_3 \wedge G_4 . \tag{2.2}$$

The equations of motion of eleven-dimensional supergravity imply that $dG_7 = 0$. However G_7 is not gauge invariant under $\delta C_3 = d\Lambda_2$. Thus S_C is not obviously gauge invariant or even local as a functional of the eleven-dimensional gauge fields. As such one should integrate by parts whenever possible and seek to find an expression which is manifestly gauge invariant.

To discuss the gauge invariance under $\delta C_3 = d\Lambda_2$ we first integrate by parts and discard all boundary terms

$$S_C = \frac{1}{3!} \epsilon^{\mu\nu\lambda} \int d^3x \left(NT_{M2} C_{\mu\nu\lambda} + \frac{3}{2} G_{\mu\nu IJ} \text{Tr}(X^I, D_\lambda X^J) - \frac{3}{2} C_{\mu IJ} \text{Tr}(X^I, \tilde{F}_{\nu\lambda} X^J) - c G_{\mu\nu\lambda IJKL} \text{Tr}(X^I, [X^J, X^K, X^L]) \right) .$$

Here we have used the fact that $C_{\mu\nu I}$ and $C_{\mu\nu\lambda IJK}$ have been projected out by the orbifold and hence $G_{\mu\nu IJ} = 2\partial_{[\mu} C_{\nu]IJ}$ and $G_{\mu\nu\lambda IJKL} = 3\partial_{[\mu} C_{\nu\lambda]IJKL}$.

We find a coupling to the world-volume gauge field strength $\tilde{F}_{\nu\lambda}$ but this term is not invariant under the gauge transformation $\delta C_3 = d\Lambda_2$. However it can be cancelled by

¹This is an assumption since the overall centre of mass zero mode x^μ that appears in (1.1) is absent in the non-Abelian generalizations.

adding the term

$$S_F = \frac{1}{4} \epsilon^{\mu\nu\lambda} \int d^3x \operatorname{Tr}(X^I, \tilde{F}_{\mu\nu} X^J) C_{\lambda IJ}, \quad (2.3)$$

to S_C . Such terms involving the world-volume gauge field strength also arise in the action of multiple D-branes.

Next consider the terms on the third line. Although G_7 is not gauge invariant $G_7 + \frac{1}{2}C_3 \wedge G_4$ is. Thus we also add the term

$$S_{CG} = -\frac{c}{2 \cdot 3!} \epsilon^{\mu\nu\lambda} \int d^3x \operatorname{Tr}(X^I, [X^J, X^K, X^L]) (C_3 \wedge G_4)_{\mu\nu\lambda IJKL}. \quad (2.4)$$

and obtain a gauge invariant action.

To summarize we find that the total flux terms are, in the limit $T_{M2} \rightarrow \infty$,

$$\begin{aligned} S_{\text{flux}} &= S_C + S_F + S_{CG} \\ &= \frac{1}{3!} \epsilon^{\mu\nu\lambda} \int d^3x \left(NT_{M2} C_{\mu\nu\lambda} + \frac{3}{2} G_{\mu\nu IJ} \operatorname{Tr}(X^I, D_\lambda X^J) \right. \\ &\quad \left. - c \left(G_7 + \frac{1}{2} C_3 \wedge G_4 \right)_{\mu\nu\lambda IJKL} \operatorname{Tr}(X^I, [X^J, X^K, X^L]) \right). \end{aligned} \quad (2.5)$$

In section 4 we will argue that $c = 2$.

2.2 Supersymmetry

In this section we wish to supersymmetrize the flux term S_{flux} that we found above. There are also similar calculations in [22–24] where the flux-induced Fermion masses on D-branes were obtained. Here we will be interested in the final term since only it preserves 3-dimensional Lorentz invariance (the first term is just a constant if it is Lorentz invariant). Thus for the rest of this section we will consider backgrounds where

$$\mathcal{L}_{\text{flux}} = c \tilde{G}_{IJKL} \operatorname{Tr}(X^I, [X^J, X^K, X^L]), \quad (2.6)$$

with

$$\begin{aligned} \tilde{G}_{IJKL} &= -\frac{1}{3!} \epsilon^{\mu\nu\lambda} \left(G_7 + \frac{1}{2} C_3 \wedge G_4 \right)_{\mu\nu\lambda IJKL} \\ &= \frac{1}{4!} \epsilon_{IJKLMNPQ} G^{MNPQ} \end{aligned} \quad (2.7)$$

and G_{IJKL} is assumed to be constant.

To proceed we take the ansatz for the Lagrangian in the presence of background fields to be

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=8} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{flux}}, \quad (2.8)$$

where $\mathcal{L}_{\mathcal{N}=8}$ is the Lagrangian detailed in [4],

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} m^2 \delta_{IJ} \operatorname{Tr}(X^I, X^J) + b \operatorname{Tr}(\bar{\Psi} \Gamma^{IJKL}, \Psi) \tilde{G}_{IJKL}, \quad (2.9)$$

and m^2 and b are constants. We use conventions where Ψ and ϵ are eleven-dimensional spinors satisfying the constraints $\Gamma_{012}\Psi = -\Psi$ and $\Gamma_{012}\epsilon = \epsilon$.

As shown in [4], $\mathcal{L}_{\mathcal{N}=8}$ is invariant under the supersymmetry transformations

$$\begin{aligned}\delta X_a^I &= i\bar{\epsilon}\Gamma^I\Psi_a \\ \delta\tilde{A}_\mu^b{}_a &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X_c^I\Psi_d f^{cdb}{}_a \\ \delta\Psi_a &= D_\mu X_a^I\Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}X_b^I X_c^J X_d^K f^{bcd}{}_a\Gamma^{IJK}\epsilon.\end{aligned}\tag{2.10}$$

We propose additional supersymmetry transformations of the following form

$$\begin{aligned}\delta' X_a^I &= 0 \\ \delta'\tilde{A}_\mu^b{}_a &= 0 \\ \delta'\Psi_a &= \omega\Gamma^{IJKL}\Gamma^M\epsilon X_a^M\tilde{G}_{IJKL},\end{aligned}\tag{2.11}$$

where ω is a real dimensionless parameter.

Applying the supersymmetry transformations to the mass deformed Lagrangian gives

$$\begin{aligned}\tilde{\delta}\mathcal{L} &= (\delta' + \delta)(\mathcal{L}_{\mathcal{N}=8} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{flux}}) \\ &= (i\omega + 2b)\text{Tr}(\bar{\Psi}\Gamma^\mu\Gamma^{MNOP}\Gamma^I\epsilon, D_\mu X^I)\tilde{G}_{MNOP} \\ &\quad + \frac{i\omega}{2}\text{Tr}(\bar{\Psi}\Gamma^{IJ}\Gamma^{MNOP}\Gamma^K\epsilon, [X^I, X^J, X^K])\tilde{G}_{MNOP} \\ &\quad - \frac{2b}{6}\text{Tr}(\bar{\Psi}\Gamma^{MNOP}\Gamma^{IJK}\epsilon, [X^I, X^J, X^K])\tilde{G}_{MNOP} \\ &\quad + 4ic\text{Tr}(\bar{\Psi}\Gamma^I\epsilon, [X^J, X^K, X^L])\tilde{G}_{IJKL} \\ &\quad + im^2\delta_{IJ}\text{Tr}(\bar{\Psi}\Gamma^I\epsilon, X^J) \\ &\quad + 2b\omega\text{Tr}(\bar{\Psi}\Gamma^{IJKL}\Gamma^{MNOP}\Gamma^Q\epsilon, X^Q)\tilde{G}_{IJKL}\tilde{G}_{MNOP}.\end{aligned}\tag{2.12}$$

To eliminate the term involving the covariant derivative we must set $b = -i\omega/2$. Substituting for b , expanding out the gamma matrices and using anti-symmetry of the indices yields

$$\begin{aligned}\tilde{\delta}\mathcal{L} &= \frac{2i\omega}{3}\text{Tr}(\bar{\Psi}\Gamma^{IJKMNOP}\epsilon, [X^I, X^J, X^K])\tilde{G}_{MNOP} \\ &\quad + (4ic - 16i\omega)\text{Tr}(\bar{\Psi}\Gamma^L\epsilon, [X^I, X^J, X^K])\tilde{G}_{LIJK} \\ &\quad + im^2\delta_{IJ}\text{Tr}(\bar{\Psi}\Gamma^I\epsilon, X^J) \\ &\quad - i\omega^2\text{Tr}(\bar{\Psi}\Gamma^{JKLM}\Gamma^{NOPQ}\Gamma^I\epsilon, X^I)\tilde{G}_{JKLM}\tilde{G}_{NOPQ}.\end{aligned}\tag{2.13}$$

Defining $\tilde{\mathcal{G}} = \tilde{G}_{JKLM}\Gamma^{JKLM}$ and using Hodge duality of the gamma matrices leads to

$$\begin{aligned}\tilde{\delta}\mathcal{L} &= \frac{96i\omega}{6}\left(-1 + \frac{c}{4\omega} - \star\right)\tilde{G}_{LIJK}\text{Tr}(\bar{\Psi}\Gamma^L\epsilon, [X^I, X^J, X^K]) \\ &\quad + i\text{Tr}(\bar{\Psi}\left(m^2 - \omega^2\tilde{\mathcal{G}}\tilde{\mathcal{G}}\right)\Gamma^I\epsilon, X^I).\end{aligned}\tag{2.14}$$

Invariance then follows if the following equations hold

$$\left(-1 + \frac{c}{4\omega} - \star\right)\tilde{G}_{LIJK} = 0 \quad \text{and} \quad \left(m^2 - \omega^2\tilde{\mathcal{G}}\tilde{\mathcal{G}}\right)\Gamma^I\epsilon = 0.\tag{2.15}$$

Since we assume that $c \neq 0$, the first equation implies $\omega = c/8$ and \tilde{G} is self-dual. It follows from the result $\Gamma^{3456789(10)}\tilde{\mathcal{G}} = \tilde{\mathcal{G}}$ that the second equation is satisfied by

$$\tilde{\mathcal{G}}\tilde{\mathcal{G}} = \frac{32m^2}{c^2} \left(1 + \Gamma^{3456789(10)}\right). \quad (2.17)$$

Expanding out the left hand side and using the self-duality of \tilde{G} one sees that this is equivalent to the two conditions

$$m^2 = \frac{c^2}{32 \cdot 4!} G^2 \quad \text{and} \quad G_{MN[IJ}G_{KL]}^{MN} = 0, \quad (2.18)$$

where $G^2 = G_{IJKL}G^{IJKL}$.

The superalgebra can be shown to close on-shell. We first consider the gauge field and find that the transformations close into the same translation and gauge transformation as in the un-deformed theory;

$$\begin{aligned} [\tilde{\delta}_1, \tilde{\delta}_2]\tilde{A}_\mu{}^b{}_a &= [\delta_1 + \delta'_1, \delta_2 + \delta'_2]\tilde{A}_\mu{}^b{}_a \\ &= v^\nu \tilde{F}_{\mu\nu}{}^b{}_a + D_\mu \tilde{\Lambda}^b{}_a, \end{aligned} \quad (2.19)$$

where $v^\nu = -2i\bar{\epsilon}_2\Gamma^\nu\epsilon_1$ and $\tilde{\Lambda}^b{}_a = i\bar{\epsilon}_2\Gamma_{JK}\epsilon_1 X_c^J X_d^K f^{cdb}{}_a$.

In considering the scalars we find a term, $2i\omega\bar{\epsilon}_2\Gamma^{MNOPIJ}\epsilon_1 X_a^J \tilde{G}_{MNOP}$, which can be transformed into an object with two gamma matrix indices by utilizing the self-duality of the flux. We find that the scalars close into a translation plus a gauge transformation and an SO(8) R-symmetry,

$$\begin{aligned} [\tilde{\delta}_1, \tilde{\delta}_2]X_a^I &= [\delta_1 + \delta'_1, \delta_2 + \delta'_2]X_a^I \\ &= v^\mu D_\mu X_a^I + \tilde{\Lambda}^b{}_a X_b^I + iR^I{}_J X_a^J, \end{aligned} \quad (2.20)$$

where $R^I{}_J = 48\omega\bar{\epsilon}_2\Gamma^{MN}\epsilon_1 \tilde{G}_{MNIJ}$ is the R-symmetry.

Finally we examine the closure of the Fermions. We find again a term incorporating $\Gamma^{(6)}$ which can be converted to $\Gamma^{(2)}$ using self-duality of \tilde{G} . Continuing, we find

$$[\tilde{\delta}_1, \tilde{\delta}_2]\Psi_a = [\delta_1 + \delta'_1, \delta_2 + \delta'_2]\Psi_a \quad (2.21)$$

$$\begin{aligned} &= v^\mu D_\mu \Psi_a + \tilde{\Lambda}^b{}_a \Psi_b + i(\bar{\epsilon}_2\Gamma_\mu\epsilon_1)\Gamma^\mu E'_\Psi - \frac{i}{4}(\bar{\epsilon}_2\Gamma_{JK}\epsilon_1)\Gamma^{JK} E'_\Psi \\ &\quad + \frac{i}{4}R_{MN}\Gamma^{MN}\Psi_a. \end{aligned} \quad (2.22)$$

Here E'_Ψ is the mass deformed Fermionic equation of motion,

$$E'_\Psi = \Gamma^\nu D_\nu \Psi_a + \frac{1}{2}\Gamma_{IJ}X_c^I X_d^J \Psi_b f^{cdb}{}_a - \omega\Gamma^{MNOP}\Psi_a \tilde{G}_{MNOP}. \quad (2.23)$$

Consequently, we find that on-shell

$$[\tilde{\delta}_1, \tilde{\delta}_2]\Psi_a = v^\mu D_\mu \Psi_a + \tilde{\Lambda}^b{}_a \Psi_b + \frac{i}{4}R_{MN}\Gamma^{MN}\Psi_a. \quad (2.24)$$

We also verify that the Fermionic equation of motion maps to the Bosonic equations of motion under the supersymmetry transformations. From the proposed mass deformed Lagrangian the scalar equation of motion is

$$E'_X = D^2 X_a^I - \frac{i}{2} \bar{\Psi}_c \Gamma^{IJ} X_d^J \Psi_b f^{cdb} - \frac{\partial V}{\partial X^I a} - m^2 X_a^I - 4c X_c^J X_d^K X_b^L f^{cdb} \tilde{G}_{IJKL} = 0. \quad (2.25)$$

The equation of motion for the gauge field is unchanged and is given by

$$E'_A = \tilde{F}_{\mu\nu}{}^b{}_a + \varepsilon_{\mu\nu\lambda} (X_c^J D^\lambda X_d^J + \frac{i}{2} \bar{\Psi}_c \Gamma^\lambda \Psi_d) f^{cdb}{}_a = 0. \quad (2.26)$$

Taking the variation of the Fermionic equation of motion (2.23) gives

$$\begin{aligned} 0 &= \Gamma^I \Gamma_\lambda X_b^I E'_A \epsilon + \Gamma^I E'_X \epsilon \\ &+ \frac{96i\omega}{6} \left(-1 + \frac{c}{4\omega} - \star \right) \tilde{G}_{LIJK} \Gamma^L \epsilon X_c^I X_d^J X_b^K f^{cdb}{}_a \\ &+ \left(m^2 - \omega^2 \Gamma^{MNOP} \Gamma^{WXYZ} \tilde{G}_{WXYZ} \tilde{G}_{MNOP} \right) \Gamma^I \epsilon X_a^I. \end{aligned} \quad (2.27)$$

Therefore consistency of the equations of motion under supersymmetry again implies that the conditions (2.16) must be satisfied.

Let us summarize our results. The Lagrangian

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} \text{Tr}(D_\mu X^I, D^\mu X^I) + \frac{i}{2} \text{Tr}(\bar{\Psi} \Gamma^\mu, D_\mu \Psi) + \frac{i}{4} \text{Tr}(\bar{\Psi} \Gamma_{IJ}, [X^I, X^J], \Psi) \\ &- V - \mathcal{L}_{CS} - \frac{1}{2} m^2 \delta_{IJ} \text{Tr}(X^I, X^J) - \frac{ic}{16} \text{Tr}(\bar{\Psi} \Gamma^{IJKL}, \Psi) \tilde{G}_{IJKL} \\ &+ c \text{Tr}([X^I, X^J, X^K], X^L) \tilde{G}_{IJKL} \end{aligned} \quad (2.28)$$

is invariant under the supersymmetries

$$\begin{aligned} \delta X_a^I &= i\bar{\epsilon} \Gamma^I \Psi_a \\ \delta \tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon} \Gamma_\mu \Gamma_I X_c^I \Psi_d f^{cdb}{}_a \\ \delta \Psi_a &= D_\mu X_a^I \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X_b^I X_c^J X_d^K f^{bcd}{}_a \Gamma^{IJK} \epsilon + \frac{c}{8} \Gamma^{IJKL} \Gamma^M \epsilon X_a^M \tilde{G}_{IJKL} \end{aligned} \quad (2.29)$$

provided \tilde{G}_{IJKL} is self-dual and satisfies (2.18). Moreover the supersymmetry algebra closes according to

$$\begin{aligned} [\delta_1, \delta_2] \tilde{A}_\mu{}^b{}_a &= v^\nu \tilde{F}_{\mu\nu}{}^b{}_a + D_\mu \tilde{\Lambda}^b{}_a \\ [\delta_1, \delta_2] X_a^I &= v^\mu D_\mu X_a^I + \tilde{\Lambda}^b{}_a X_b^I + i R^I{}_J X_a^J \\ [\delta_1, \delta_2] \Psi_a &= v^\mu D_\mu \Psi_a + \tilde{\Lambda}^b{}_a \Psi_b + \frac{i}{4} R_{MN} \Gamma^{MN} \Psi_a. \end{aligned} \quad (2.30)$$

Taking

$$G = \mu(dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^6 + dx^7 \wedge dx^8 \wedge dx^9 \wedge dx^{10}) \quad (2.31)$$

readily leads to the mass-deformed Lagrangian of [13, 14].

3 $\mathcal{N} = 6$ Theories

Let us now consider the more general case of $\mathcal{N} = 6$ supersymmetry and in particular the ABJM [10] and ABJ [11] models which describe an arbitrary number of M2-branes in an $\mathbb{R}^8/\mathbb{Z}_k$ orbifold. We will use the notation and conventions of [25]. Since the discussion is similar in spirit to the $\mathcal{N} = 8$ case we will shorten our discussion and largely just present the results of our calculations.

3.1 Non-Abelian couplings to background fluxes

In the $\mathcal{N} = 6$ theories there are 4 complex scalars Z^A and their complex conjugates \bar{Z}_A . These are defined in terms of the spacetime coordinates through

$$\begin{aligned} Z^1 &= \frac{1}{\sqrt{2T_{M2}}}(x^3 + ix^4) & Z^2 &= \frac{1}{\sqrt{2T_{M2}}}(x^5 + ix^6) \\ Z^3 &= \frac{1}{\sqrt{2T_{M2}}}(x^7 - ix^9) & Z^4 &= \frac{1}{\sqrt{2T_{M2}}}(x^8 - ix^{10}) . \end{aligned}$$

In particular we will take the formulation in [25]. The scalars and Fermions are endowed with a triple product $[Z^A, Z^B; \bar{Z}_C]$ or $[\bar{Z}_A, \bar{Z}_B; Z^C]$ and an inner-product $\text{Tr}(\bar{Z}_A, Z^B)$ subject to a quadratic fundamental identity as well as the condition $\text{Tr}(\bar{Z}_D, [Z^A, Z^B; \bar{Z}_C])^* = -\text{Tr}(\bar{Z}_A, [Z^C, Z^D; \bar{Z}_B])$. To obtain the ABJM/ABJ models [10, 11] one should let the fields be $m \times n$ matrices and define

$$[Z^A, Z^B; \bar{Z}_C] = \lambda(Z^A \bar{Z}_C^\dagger Z^B - Z^B \bar{Z}_C^\dagger Z^A) . \quad (3.1)$$

where λ is an arbitrary (but quantized) coupling constant. As such the gauge invariant terms always involve an equal number of Z and \bar{Z} coordinates. Again this is consistent with the interpretation that the M2-branes are in an $\mathbb{C}^4/\mathbb{Z}_k$ orbifold which acts as $Z^A \rightarrow e^{\frac{2\pi i}{k}} Z^A$.

Following the discussion of the previous section we start with

$$\begin{aligned} S_C &= \frac{1}{3!} \epsilon^{\mu\nu\lambda} \int d^3 x \left(NT_{M2} C_{\mu\nu\lambda} + \frac{3}{2} C_{\mu}{}^A{}_B \text{Tr}(D_\nu \bar{Z}_A, D_\lambda Z^B) \right. \\ &\quad + \frac{3}{2} C_{\mu A}{}^B \text{Tr}(D_\nu Z^A, D_\lambda \bar{Z}_B) \\ &\quad + \frac{3c}{2} C_{\mu\nu AB}{}^{CD} \text{Tr}([D_\lambda \bar{Z}_D, [Z^A, Z^B; \bar{Z}_C]]) \\ &\quad \left. + \frac{3c}{2} C_{\mu\nu}{}^{AB}{}_{CD} \text{Tr}([D_\lambda Z^D, [\bar{Z}_A, \bar{Z}_B; Z^C]]) \right) . \quad (3.2) \end{aligned}$$

Integrating by parts we again find a non-gauge invariant term proportional to $\epsilon^{\mu\nu\lambda} \tilde{F}_{\nu\lambda} C_{\mu}{}^A{}_B$ which is cancelled by adding

$$S_F = \frac{1}{8} \epsilon^{\mu\nu\lambda} \int d^3 x C_{\mu}{}^A{}_B \text{Tr}(\bar{Z}_A, \tilde{F}_{\nu\lambda} Z^B) + C_{\mu A}{}^B \text{Tr}(Z^A, \tilde{F}_{\nu\lambda} \bar{Z}_B) . \quad (3.3)$$

As with the case above we also must add

$$S_{CG} = -\frac{c}{8 \cdot 3!} \epsilon^{\mu\nu\lambda} \int d^3 x (C_3 \wedge G_4)_{\mu\nu AB}{}^{CD} \text{Tr}(\bar{Z}_D, [Z^A, Z^B; \bar{Z}_C]) \quad (3.4)$$

to ensure that the last term is gauge invariant. Thus in total we have

$$\begin{aligned}
 S_{\text{flux}} &= S_C + S_F + S_{CG} \\
 &= \frac{1}{3!} \epsilon^{\mu\nu\lambda} \int d^3x \left(NT_{M2} C_{\mu\nu\lambda} + \frac{3}{4} G_{\mu\nu}{}^A{}_B \text{Tr}(\bar{Z}_A, D_\lambda Z^B) + \frac{3}{4} G_{\mu\nu A}{}^B \text{Tr}(Z^A, D_\lambda \bar{Z}_B) \right. \\
 &\quad \left. - \frac{c}{4} (G_7 + \frac{1}{2} C_3 \wedge G_4)_{\mu\nu\lambda AB}{}^{CD} \text{Tr}([\bar{Z}_D, [Z^A, Z^B; \bar{Z}_C]]) \right). \quad (3.5)
 \end{aligned}$$

3.2 Supersymmetry

Following on as before we wish to supersymmetrize the action

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=6} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{flux}}, \quad (3.6)$$

where $\mathcal{L}_{\mathcal{N}=6}$ is the $\mathcal{N} = 6$ Chern-Simons-Matter Lagrangian. We restrict to backgrounds where

$$\mathcal{L}_{\text{flux}} = \frac{c}{4} \text{Tr}([\bar{Z}_D, [Z^A, Z^B; \bar{Z}_C]]) \tilde{G}_{AB}{}^{CD}, \quad (3.7)$$

with

$$\begin{aligned}
 \tilde{G}_{AB}{}^{CD} &= -\frac{1}{3!} \epsilon^{\mu\nu\lambda} \left(G_7 + \frac{1}{2} C_3 \wedge G_4 \right)_{\mu\nu\lambda AB}{}^{CD} \\
 &= \frac{1}{4} \epsilon_{ABEF} \epsilon^{CDGH} G^{EF}{}_{GH}. \quad (3.8)
 \end{aligned}$$

Finally we take the ansatz for $\mathcal{L}_{\text{mass}}$ to be

$$\mathcal{L}_{\text{mass}} = -m^2 \text{Tr}(\bar{Z}_A, Z^A) + b \text{Tr}(\bar{\psi}^A, \psi_F) \tilde{G}_{AE}{}^{EF}. \quad (3.9)$$

We propose the following modification to the Fermion supersymmetry variation

$$\delta' \psi_{Ad} = \omega \epsilon_{DF} Z_d^F \tilde{G}_{AE}{}^{ED}, \quad (3.10)$$

where ω is a real parameter.

After applying the supersymmetry transformations to \mathcal{L} we find that taking $b = -i\omega$ eliminates the covariant derivative terms. The terms that are second order in \tilde{G} must vanish separately and this gives the condition

$$\tilde{G}_{AE}{}^{EB} \tilde{G}_{BF}{}^{FC} = \frac{m^2}{\omega^2} \delta_A^C. \quad (3.11)$$

The remaining terms in the variation are

$$\begin{aligned}
 \delta \mathcal{L} &= +2i\omega \text{Tr}(\bar{Z}_D, [\bar{\psi}_F \epsilon^{DA}, Z^Q; \bar{Z}_Q]) \tilde{G}_{AE}{}^{EF} \\
 &\quad + i\omega \text{Tr}(\bar{Z}_D, [\bar{\psi}_F \epsilon^{QD}, Z^A; \bar{Z}_Q]) \tilde{G}_{AE}{}^{EF} \\
 &\quad + 2i\omega \text{Tr}(\bar{Z}_D, [\bar{\psi}_K \epsilon^{AD}, Z^K; \bar{Z}_F]) \tilde{G}_{AE}{}^{EF} \\
 &\quad + \frac{ic}{2} \text{Tr}(\bar{Z}_D, [\bar{\psi}_K \epsilon^{AK}, Z^B; \bar{Z}_C]) \tilde{G}_{AB}{}^{CD} \\
 &\quad + \frac{i\omega}{2} \epsilon^{AKQD} \epsilon_{IJFP} \text{Tr}(\bar{Z}_D, [\bar{\psi}_K \epsilon^{IJ}, Z^P; \bar{Z}_Q]) \tilde{G}_{AE}{}^{EF} \\
 &\quad + \text{c.c.}, \quad (3.12)
 \end{aligned}$$

where we have made use of the reality condition $\epsilon_{FP} = \frac{1}{2}\epsilon_{IJFP}\epsilon^{IJ}$. To proceed we need to restrict \tilde{G} to have the form

$$\tilde{G}_{AB}{}^{CD} = \frac{1}{2}\delta_B^C\tilde{G}_{AE}{}^{ED} - \frac{1}{2}\delta_A^C\tilde{G}_{BE}{}^{ED} - \frac{1}{2}\delta_B^D\tilde{G}_{AE}{}^{EC} + \frac{1}{2}\delta_A^D\tilde{G}_{BE}{}^{EC}, \quad (3.13)$$

with $\tilde{G}_{AE}{}^{EA} = 0$. Substituting for $\tilde{G}_{AB}{}^{CD}$ allows us to factor out the common term $\text{Tr}(\bar{Z}_D, [\bar{\psi}_K\epsilon^{IJ}, Z^P; \bar{Z}_Q])\tilde{G}_{AE}{}^{EF}$. This factor is separately anti-symmetric in IJ and DQ so after expanding out $\epsilon^{AQKD}\epsilon_{IJFP} = 4!\delta_{IJFP}^{[AQKD]}$ we have

$$\begin{aligned} \delta\mathcal{L} &= i\omega \left(\frac{c}{2\omega} - 2 \right) (\delta_I^A\delta_J^K\delta_F^D\delta_P^Q + \delta_I^K\delta_J^Q\delta_F^D\delta_P^A) \\ &\quad \times \text{Tr}(\bar{Z}_D, [\bar{\psi}_K\epsilon^{IJ}, Z^P; \bar{Z}_Q])\tilde{G}_{AE}{}^{EF} \\ &\quad + \text{c.c.} \end{aligned} \quad (3.14)$$

Therefore the Lagrangian is invariant under supersymmetry if $\omega = c/4$. Taking the trace of equation (3.11) allows us to deduce that

$$m^2 = \frac{1}{32 \cdot 4!}c^2G^2 \quad (3.15)$$

where $G^2 = 6G_{AB}{}^{CD}G^{AB}{}_{CD} = 12G_{AE}{}^{EB}G_{BF}{}^{FA}$.

In examining the closure of the superalgebra we find

$$[\delta_1, \delta_2] \tilde{A}_\mu{}^c{}_d = v^\nu \tilde{F}_{\mu\nu}{}^c{}_d + D_\mu(\Lambda_{\bar{a}b}f^{cb\bar{a}}{}_d) \quad (3.16)$$

$$[\delta_1, \delta_2] Z_d^A = v^\mu D_\mu Z_d^A + \Lambda_{\bar{c}b}f^{ab\bar{c}}{}_d Z_a^A - iR^A{}_B Z_d^B - iY Z_d^A \quad (3.17)$$

where

$$v^\mu = \frac{i}{2}\bar{\epsilon}_2^{CD}\gamma^\mu\epsilon_{CD}^1 \quad (3.18)$$

$$\Lambda_{\bar{c}b} = i(\bar{\epsilon}_2^{DE}\epsilon_{CE}^1 - \bar{\epsilon}_1^{DE}\epsilon_{CE}^2)\bar{Z}_{D\bar{c}}Z_b^C \quad (3.19)$$

$$R^A{}_B = \omega \left((\bar{\epsilon}_1^{AC}\epsilon_{DB}^2 - \bar{\epsilon}_2^{AC}\epsilon_{DB}^1) - \frac{1}{4}(\bar{\epsilon}_1^{EC}\epsilon_{DE}^2 - \bar{\epsilon}_2^{EC}\epsilon_{DE}^1)\delta_B^A \right) \tilde{G}_{CM}{}^{MD} \quad (3.20)$$

$$Y = \frac{\omega}{4}(\bar{\epsilon}_1^{EC}\epsilon_{DE}^2 - \bar{\epsilon}_2^{EC}\epsilon_{DE}^1)\tilde{G}_{CM}{}^{MD}. \quad (3.21)$$

Acting with the commutator on the Fermions gives

$$\begin{aligned} [\delta_1, \delta_2]\psi_{Dd} &= v^\mu D_\mu\psi_{Dd} + \Lambda_{\bar{a}b}f^{cb\bar{a}}{}_d\psi_{Dc} \\ &\quad - \frac{i}{2}(\bar{\epsilon}_1^{AC}\epsilon_{2AD} - \bar{\epsilon}_2^{AC}\epsilon_{1AD})E'_{Cd} \\ &\quad + \frac{i}{4}(\bar{\epsilon}_1^{AB}\gamma_\nu\epsilon_{2AB})\gamma^\nu E'_{Dd} \\ &\quad + iR^A{}_D\psi_{Ad} - iY\psi_{Dd} \end{aligned} \quad (3.22)$$

provided the 4-form satisfies $\tilde{G}_{AE}{}^{EA} = 0$. The new Fermionic equation of motion is

$$\begin{aligned} E'_{Cd} &= \gamma^\mu D_\mu\psi_{Cd} + f^{ab\bar{c}}{}_d\psi_{Ca}Z_b^D\bar{Z}_{D\bar{c}} - 2f^{ab\bar{c}}{}_d\psi_{Da}Z_b^D\bar{Z}_{C\bar{c}} \\ &\quad - \epsilon_{CDEF}f^{ab\bar{c}}{}_d\psi_{\bar{c}}^D Z_a^E Z_b^F + \frac{c}{4}\tilde{G}_{CE}{}^{EB}\psi_{Bd}. \end{aligned} \quad (3.23)$$

Consistency of the Bosonic and Fermionic equations of motion under supersymmetry requires that $\tilde{G}_{AE}{}^{EB}\tilde{G}_{BF}{}^{FC} = \frac{m^2}{\omega^2}\delta_A^C$, which is the same condition as found in demonstrating invariance of the action.

Choosing $\tilde{G}_{AB}{}^{CD}$ to have the form (3.13) with

$$\tilde{G}_{AB}{}^{BC} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & -\mu \end{pmatrix}, \tag{3.24}$$

gives the mass-deformed Lagrangian of [15, 16].

4 Background curvature

Our final point is to understand the physical origin of the mass-squared term in the effective action which is quadratic in the masses. Note that this term is a simple, SO(8)-invariant mass term for all the scalar fields. Furthermore it does not depend on any non-Abelian features of the theory. Therefore we can derive this term by simply considering a single M2-brane and compute the unknown constant c .

We can understand the origin of this term as follows. We have seen that it arises as a consequence of supersymmetry. For a single M2-brane supersymmetry arises as a consequence of κ -symmetry and κ -symmetry is valid whenever an M2-brane is propagating in a background that satisfies the equations of motion of eleven-dimensional supergravity [1].

The multiple M2-brane actions implicitly assume that the background is simply flat space or an orbifold thereof. However the inclusion of a non-trivial flux implies that there is now a source for the eleven-dimensional metric which is of order flux-squared. Thus for there to be κ -supersymmetry and hence supersymmetry it follows that the background must be curved. This in turn will lead to a potential in the effective action of an M2-brane. In particular given a 4-form flux G_4 the Bosonic equations of eleven-dimensional supergravity are

$$\begin{aligned} R_{mn} - \frac{1}{2}g_{mn}R &= \frac{1}{2 \cdot 3!}G_{mpqr}G_n{}^{pqr} - \frac{1}{4 \cdot 4!}g_{mn}G^2 \\ d \star G_4 - \frac{1}{2}G_4 \wedge G_4 &= 0. \end{aligned} \tag{4.1}$$

At lowest order in fluxes we see that $g_{mn} = \eta_{mn}$ and G_4 is constant. However at second order there are source terms. To start with we will assume that, at lowest order, only G_{IJKL} is non-vanishing. To solve these equations we introduce a non-trivial metric of the form

$$g_{mn} = \begin{pmatrix} e^{2\omega}\eta_{\mu\nu} & 0 \\ 0 & g_{IJ} \end{pmatrix}, \tag{4.2}$$

where $\omega = \omega(x^I) = \omega(X^I/T_{M2}^{\frac{1}{2}})$ and $g_{IJ} = g_{IJ}(x^I) = g_{IJ}(X^I/T_{M2}^{\frac{1}{2}})$.

Let us look at an M2-brane in this background. The first term in the action (1.1) is

$$\begin{aligned}
 S_1 &= -T_{M2} \int d^3x \sqrt{-\det(e^{2\omega}\eta_{\mu\nu} + \partial_\mu x^I \partial_\nu x^J g_{IJ})} \\
 &= -T_{M2} \int d^3x e^{3\omega} \left(1 + \frac{1}{2} e^{-2\omega} \partial_\mu x^I \partial^\mu x^J g_{IJ} + \dots \right) \\
 &= - \int d^3x \left(T_{M2} e^{3\omega} + \frac{1}{2} e^\omega \partial_\mu X^I \partial^\mu X^J g_{IJ} + \dots \right).
 \end{aligned} \tag{4.3}$$

Next we note that, in the decoupling limit $T_{M2} \rightarrow \infty$, we can expand

$$e^{2\omega(x)} = e^{2\omega(X^I/\sqrt{T_{M2}})} = 1 + \frac{2}{T_{M2}} \omega_{IJ} X^I X^J + \dots, \tag{4.4}$$

and

$$g_{IJ}(x) = g_{IJ}(X^I/\sqrt{T_{M2}}) = \delta_{IJ} + \dots, \tag{4.5}$$

so that

$$S_1 = - \int d^3x \left(T_{M2} + 3\omega_{IJ} X^I X^J + \frac{1}{2} \partial_\mu X^I \partial^\mu X^J \delta_{IJ} + \dots \right), \tag{4.6}$$

where the ellipsis denotes terms that vanish as $T_{M2} \rightarrow \infty$. Thus we see that in the decoupling limit we obtain the mass term for the scalars. Similar mass terms for M2-branes were also studied in [26] for pp-waves.

To compute the warp-factor ω we can expand $g_{mn} = \eta_{mn} + h_{mn}$, where h_{mn} is second order in the fluxes, and linearize the Einstein equation. If we impose the gauge $\partial^m h_{mn} - \frac{1}{2} \partial_n h^p_p = 0$ then Einstein's equation becomes

$$-\frac{1}{2} \partial_p \partial^p \left(h_{mn} - \frac{1}{2} \eta_{mn} h^q_q \right) = \frac{1}{2 \cdot 3!} G_{mpqr} G_n{}^{pqr} - \frac{1}{4 \cdot 4!} g_{mn} G^2. \tag{4.7}$$

This reduces to two coupled sets of equations corresponding to choosing indices $(m, n) = (\mu, \nu)$ and $(m, n) = (I, J)$. Contracting the latter with δ^{IJ} one finds that $h_I{}^I = 4h_p{}^p$ and hence $h_p{}^p = -\frac{1}{3}h_\mu{}^\mu$. With this in hand the $(m, n) = (\mu, \nu)$ terms in Einstein's equation reduce to

$$\partial_I \partial^I e^{2\omega} = \frac{1}{3 \cdot 4!} G^2, \tag{4.8}$$

and hence, to leading order in the fluxes,

$$e^{2\omega} = 1 + \frac{1}{48 \cdot 4!} G^2 \delta_{IJ} x^I x^J, \tag{4.9}$$

so that S_1 contributes the term

$$S_1 = - \int d^3x \frac{1}{32 \cdot 4!} G^2 X^2 \tag{4.10}$$

to the potential.

Next we must look at the second, Wess-Zumino term, in (1.1);

$$S_2 = \frac{T_{M2}}{3!} \int d^3x \epsilon^{\mu\nu\lambda} C_{\mu\nu\lambda}. \tag{4.11}$$

Although we have assumed that $C_{\mu\nu\lambda} = 0$ at leading order, the C -field equation of motion implies that $G_{I\mu\nu\lambda} = \partial_I C_{\mu\nu\lambda}$ is second order in G_{IJKL} . In particular if we write $C_{\mu\nu\lambda} = C_0 \epsilon_{\mu\nu\lambda}$ we find, assuming G_{IJKL} is self-dual, the equation

$$\partial_I \partial^I C_0 = \frac{1}{2 \cdot 4!} G^2, \tag{4.12}$$

where $G^2 = G_{IJKL} G^{IJKL}$. The solution is

$$C_0 = \frac{1}{32 \cdot 4!} G^2 \delta_{IJ} x^I x^J. \tag{4.13}$$

Thus we find that S_2 gives a second contribution to the scalar potential

$$S_2 = - \int d^3x \frac{1}{32 \cdot 4!} G^2 X^2. \tag{4.14}$$

Note that this is equal to the scalar potential derived from S_1 . Therefore if we were to break supersymmetry and consider anti-M2-branes, where the sign of the Wess-Zumino term changes, we would not find a mass for the scalars.

In total we find the mass-squared

$$m^2 = \frac{1}{8 \cdot 4!} G^2. \tag{4.15}$$

Comparing with (2.18) we see that $c^2 = 4$, e.g. $c = 2$. Note that we have performed this calculation using the notation of the $\mathcal{N} = 8$ theory, however a similar calculation also holds in the $\mathcal{N} = 6$ case with the same result.

5 Conclusions

In this paper we discussed the coupling of multiple M2-branes with $\mathcal{N} = 6, 8$ supersymmetry to the background gauge fields of eleven-dimensional supergravity. In particular we gave a local and gauge invariant form for the ‘Myers terms’ in the limit $M_{pl} \rightarrow \infty$. We supersymmetrized these flux terms in the case where the fluxes preserve the supersymmetry and Lorentz symmetry of M2-branes to obtain the massive models of [13–16]. We also showed how the flux-squared term in the effective action, which arises as a mass term for the scalar fields, is generated through a back reaction of the fluxes on the eleven-dimensional geometry.

The results we have found using gauge invariance fit naturally with the $\mathbb{R}^8/\mathbb{Z}_k$ orbifold interpretation of the background. However for the $\mathcal{N} = 6$ theories with $k = 1, 2$ the orbifold action is less restrictive and this allows for additional terms. In particular for $k = 2$ we expect terms where total number of Z^A and \bar{Z}_B fields are even (but not necessarily equal). In addition for $k = 1$ there should be terms with any number of Z^A and \bar{Z}_B fields. Such terms are not gauge invariant on their own but presumably can be made so by including monopole operators which, for $k = 1, 2$, are local.

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